The Unit Circle

Things you should already know

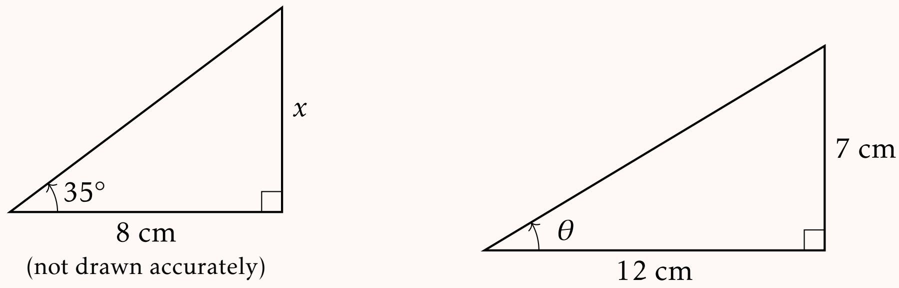
Fact (Trigonometric Ratios) — For a right-angled triangle with angle θ:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Fact (SOHCAHTOA) — **S**in = **O**pp / **H**yp, **C**os = **A**dj / **H**yp, **T**an = **O**pp / **A**dj
These definitions work for angles between 0° and 90° in a right-angled triangle.

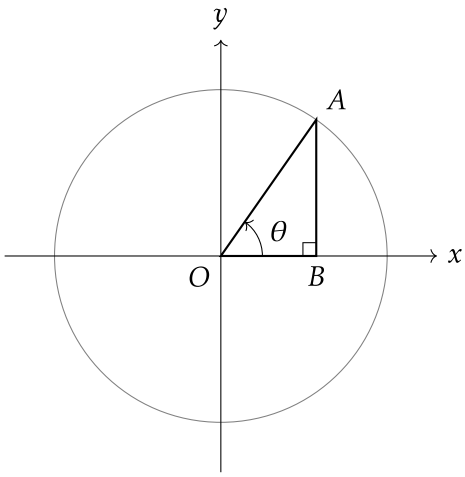
Fact (Calculator Functions) — Your calculator has sin, cos, and tan buttons — these take an angle and return the ratio.
It also has $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ (sometimes written arcsin, arccos, arctan) — these take a ratio and return the angle.

Example
Find the length x and the angle θ in the triangles below.

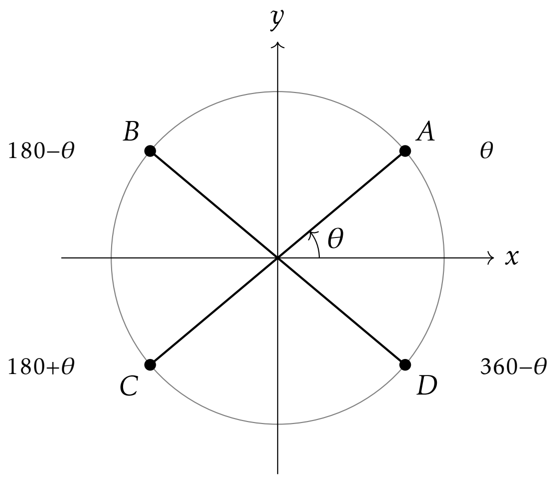


(not drawn accurately)

The Unit Circle



The Four Quadrants



Example

In which quadrants is each trig function positive?

Example

Without a calculator, determine the sign of each:

(a) $\sin 130°$ (b) $\cos 210°$ (c) $\tan 315°$

Fact (Exact Values) —

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Example

Without a calculator, calculate the value of each:

(a) $\sin 330°$ (b) $\cos 225°$ (c) $\tan 240°$

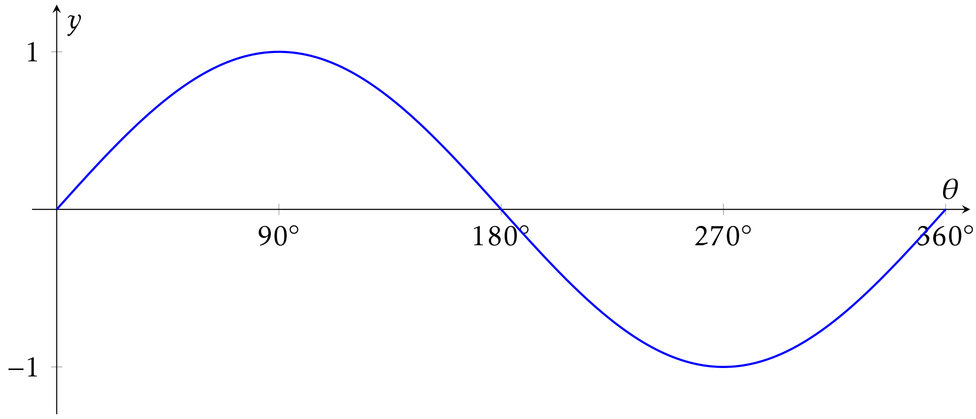
Graphs of the Trigonometric Functions

Example

Using the unit circle, fill in the table for $y = \sin\theta$:

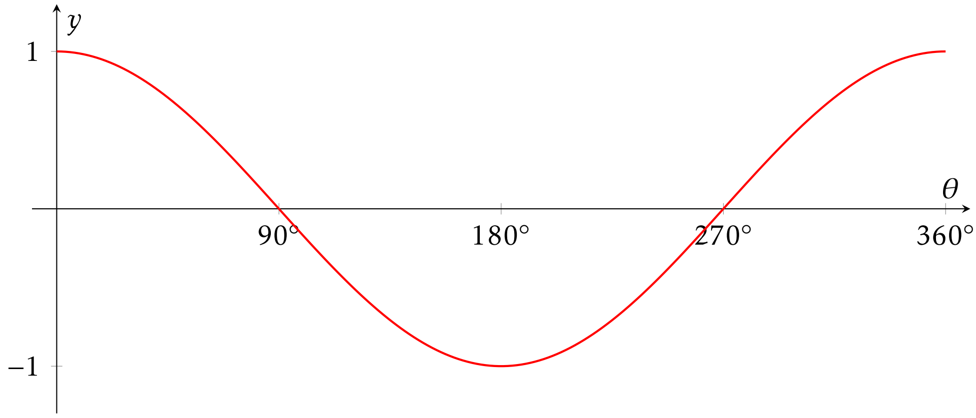
θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
$\sin\theta$													

The graph of $y = \sin\theta$



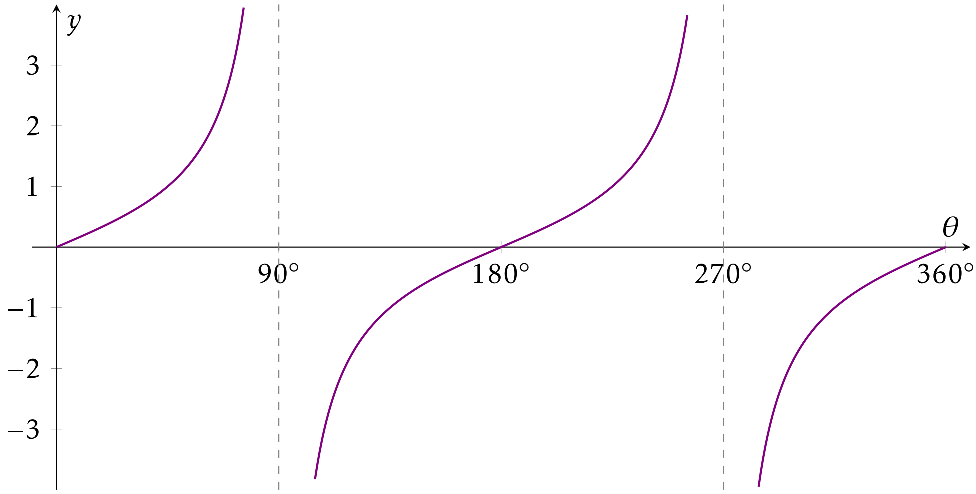
Key properties of $y = \sin\theta$:

The graph of $y = \cos\theta$



Key properties of $y = \cos\theta$:

The graph of $y = \tan\theta$



Key properties of $y = \tan\theta$:

Example

You are told that $\sin 24° = 0.407$. Without a calculator, write down the values of:

(a) $\sin 156°$ (b) $\sin 204°$ (c) $\sin 336°$

Solving Trigonometric Equations

Example

Using a sketch of $y = \sin\theta$ for $0° \leq \theta \leq 360°$, how many solutions does $\sin\theta = 0.5$ have? What about $\sin\theta = -0.3$?

Method

Fact — To solve a trigonometric equation over $0° \leq \theta \leq 360°$:

1. Use your calculator to find the **first solution** (the "principal value").
2. Use the **symmetry of the graph** to find the **second solution**.
3. Check both solutions are in the required range.

Example

Solve $\sin\theta = 0.6$ for $0° \leq \theta \leq 360°$. Give answers to 1 d.p.

Example

Solve $\cos\theta = -0.4$ for $0° \leq \theta \leq 360°$. Give answers to 1 d.p.

Example

Solve $\tan\theta = -2$ for $0° \leq \theta \leq 360°$. Give answers to 1 d.p.

Example

Solve $\sin\theta = -0.5$ for $0^\circ \le \theta \le 360^\circ$.

Extension: Equations of the form $\sin k\theta = c$

Fact — For equations like $\sin 2\theta = c$ or $\cos 3\theta = c$:

1. Let $u = k\theta$ and solve $\sin u = c$ in the range $0^\circ \le u \le 360k^\circ$
2. Divide all solutions by k to get θ

There are usually $2k$ solutions in any 360° range.

Example

Solve $\cos 2\theta = 0.5$ for $0^\circ \le \theta \le 360^\circ$.

Extension: Quadratic Trigonometric Equations

Example

Solve $2\sin^2\theta - \sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$.

Example

Solve $2\cos^2\theta - 5\cos\theta + 2 = 0$ for $0° \leq \theta \leq 360°$.